Solving Jerry Gordon’s *Giant Pyramid*

by George Bell

Introduction

In the 1970’s and 1980’s Leonard and Jerry Gordon devised many ball-pyramid puzzles [1]. One version devised by Jerry Gordon was called *Giant Pyramid*, see Figure 2. The goal of this puzzle is to make a 35-ball tetrahedron from the nine pieces in Figure 1. This particular version, marketed under the name “*Giant Pyramid*”, was reputed to be the most difficult—for one thing it has a unique solution. This puzzle is currently sold under this name by Kadon Enterprises [2], with a solution time given as “one hour to a year, or more”. Leonard Gordon called his brother’s puzzle “diabolical” [1], one wonders if the pun was intentional!

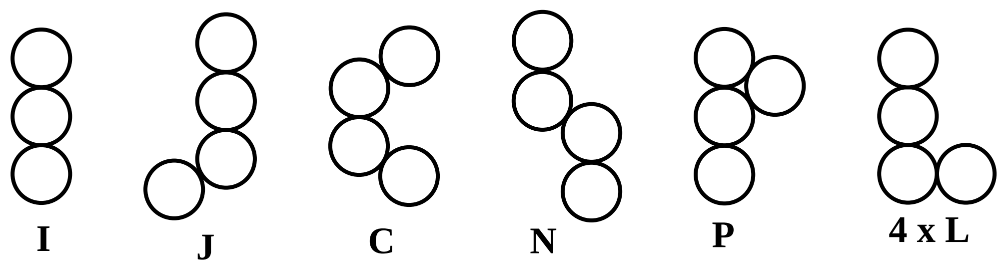

Figure 1. The nine planar pieces in The Giant Pyramid puzzle. All angles between three touching balls are 60, 90, 120, or 180 degrees.

Before reading further, I suggest you buy or make the puzzle and try it yourself.

A workable copy can be made by gluing together wood balls from a craft store. Or try marbles, styrofoam balls, ping-pong balls, oranges—whatever spheres you have in good supply.

Below I present a technique for analyzing polysphere puzzles which is quite effective for this puzzle. Of course, the puzzle can be solved in a fraction of a second by BurrTools [3], but we seek techniques useful to human solvers.

Figure 2. The original *Giant Pyramid*, made by Gordon Brothers, ca. 1986.

Puzzle geometry

The idea is to exploit the symmetry of the tetrahedron. In Figure 3 we see the layers of the assembled puzzle as it sits on a table. Each ball is labeled with a category, which represents its role in the symmetry of the tetrahedron. No matter how the tetrahedron is rotated or reflected, these labels do not change.

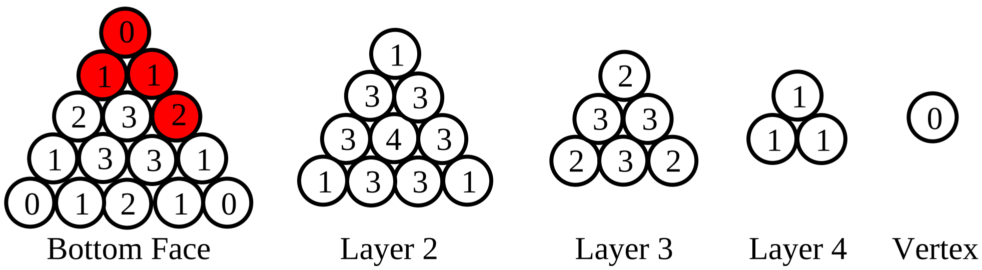

Figure 3. The 35-ball tetrahedron sliced along hexagonal packing planes. The (red) shaded balls indicate one possible placement of piece P.

All pieces except L must be placed flat in such layers (parallel to some face). The L pieces cannot be placed entirely in one layer, they must lie tilted with respect to these planes. The L pieces make the puzzle more difficult, because it is hard to visualize all the ways they can be packed. If we take a pair of opposite edges of the tetrahedron, they define a set of parallel planes, and if we slice the tetrahedron along these planes we end up with Figure 4. The L pieces must be placed in these planes.

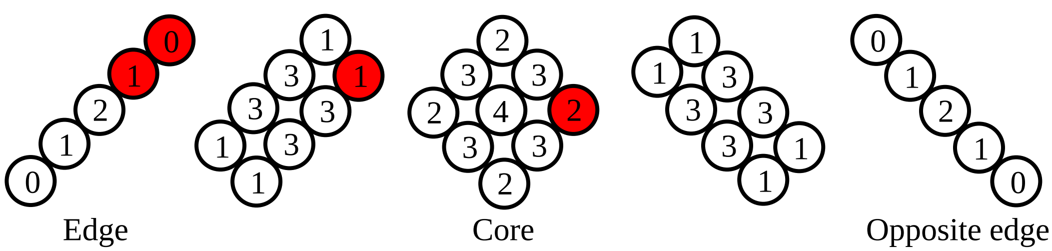

Figure 4. The same 35-ball tetrahedron sliced along square packing planes.

Category	Description	Count
0	vertex	4
1	touches a vertex	12
2	middle of an edge	6
3	face	12
4	centre	1
Total		35

Table 1. The five categories.

Table 1 shows the description and count for each of the five categories. A solution to the puzzle must cover these categories with exactly these counts. Consider, for example, the placement of the L pieces. We can see from Figure 4 that an L can never occupy a vertex, and it must use up at least two 3's. Among all four L pieces at least eight 3's are used, leaving at most four 3's for the remaining five pieces. Also, notice that no piece can occupy more than one vertex, which means that exactly four of the five pieces {I, J, C, N, P} occupy vertices, with the remaining piece being "free".

Piece	Vertex	Centre	Other
I	(012)	(343)	(121), (133) or (232)
J	(0123) or (0133)	(1343)	(1121), (1232), (1331), (2331) or (3331)
C	(0132)	(1343)	(1331), (2332), (1233), (1123), (3333) or (2112)
N	(0133)	(1343)	(1332) or (1321)
P	(0112) or (0123)	(3433) or (1343)	(1333), (1133), (2332), (3312), (2312) or (1231)
L	never	(3432)	(1133), (1333) or (2323)

Table 2. **Piece categories, if it occupies a vertex, the centre, or any other location.**

Table 2 shows the categories used by each piece depending on where it is placed in the tetrahedron. For example, consider P in the orientation indicated by the (red) shaded balls in Figure 3. This corresponds to the categories (0112) from Table 2, showing that piece P occupies one vertex (0), two 1's and one 2 (edge). The order of these numbers is unimportant, i.e. (0112) is the same as (2101).

Solving the Giant Pyramid

A useful question is: Which piece occupies the centre (4)? Suppose, for example, that piece C lies in the centre. Let us tally up the number of 3's used by the pieces. We have C in the centre (2), the 4 L's (at least 8), N at a vertex (2), J at a vertex (at least 1). The total number of 3's used is at least 13, one more than are available! Apparently, the assumption that C is in the centre is false. The same reasoning works for pieces I, J and P. We conclude that for the Giant Pyramid puzzle:

Only N or L can occupy the centre (4).

Let us consider the first possibility: N in the centre. We then use at least ten 3's among the four L's and N in the centre, leaving only two for the remaining four vertex pieces. There is only one way to place the vertex pieces that uses exactly two 3's: I (012), J (0123), C (0132), P (0112). A quick tally of the remaining categories reveals the count to be covered by the 4 L's: six 1's, two 2's and eight 3's. The only way this can be accomplished is to have three L's (1133) and the last L (2323). Remarkably, we have determined unique locations for all nine pieces!

So is the puzzle solved? Definitely not! The location of each piece has only been determined up to the symmetries of a tetrahedron. Conceivably, each piece could lie in one of up to 12 orientations within the tetrahedron. Positioning all pieces as specified could well be impossible, in which case this line of reasoning is a dead end.

To proceed further, consider the four vertex pieces: I, J, C and P. The first three cover a single 1, and the other two 1's associated with this vertex must be covered by another piece, which can only be the three L's. Piece P covers two 1's, and the remaining 1 can only be covered by N, in fact pieces P and N must lie in parallel planes. Thus, we have the following piece pairs occupying the four vertices: I+L, J+L, C+L and P+N. We can now physically combine these pieces, and try to assemble them into the tetrahedron, along with the one remaining L. We have reduced a 9 piece puzzle to a 5 piece puzzle!

The combined pieces are somewhat complex 3D shapes. Figure 5 shows the balls occupied by each of the five combined pieces, where we have "unwrapped" the tetrahedron to show all affected faces. Note that only one combined piece, P+N, affects all four faces of the tetrahedron. Each paired piece can also be constructed in (at least) two ways, so we could take the mirror image of each diagram.

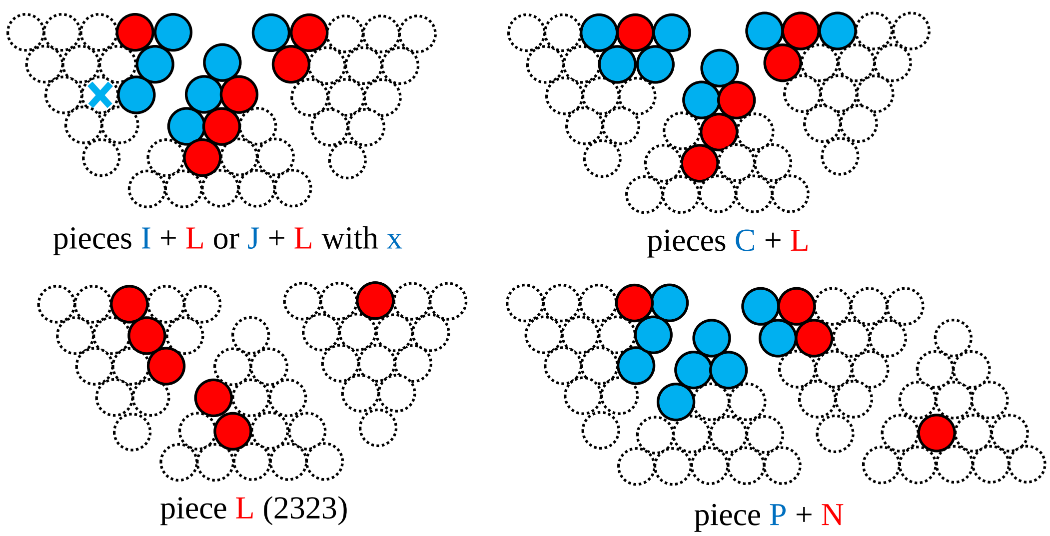

Figure 5. Faces covered by the five piece combinations.

Let us now fix the orientation of the puzzle by beginning the assembly with piece P, place it flat in the upper vertex, bottom layer (shaded red in Figure 3). We know that the remaining 1 for this vertex is filled by N, so this piece must lie in layer 2 covering the upper 1. We now must fill the rest of the bottom layer using two of the corners I+L, J+L or C+L, plus possibly the free L. There are not many ways to accomplish this, shown in Figure 6. A few other cases are not shown because they immediately lead to piece conflicts (usually with the N in the second layer).

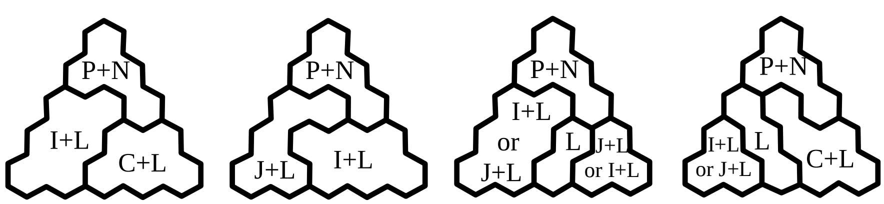

Figure 6. Ways the combined pieces can fill the bottom layer.

The reader can check that **only the rightmost diagram in Figure 6 leads to a solution**. The others are dead ends. Note that each diagram represents multiple options, because each piece combination, as well as the free L, can be oriented multiple ways. An alternate technique for discovering the solution is to fix the bottom layer as the face which does **not** involve the free L. Then this bottom layer must be filled by the three vertex pairs alone, so using one of the left two diagrams in Figure 6. This technique finds the same solution, but in a different orientation. A step by step solution (in the orientation of Figure 6) is shown in Figure 6.5.

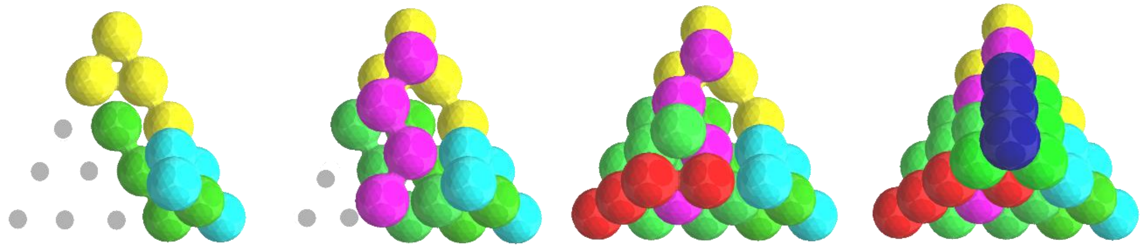

Figure 6.5. The unique solution constructed in 4 steps.

Vertices	Centre	Other L's	Free
I (012), J (0123), C (0132), P (0112)	L (3432)	3x(1133)	N (1332)
I (012), J (0123), C (0132), P (0123)	L (3432)	3x(1133)	N (1321)
I (012), J (0123), C (0132), P (0112)	L (3432)	2x(1133), (1333)	N (1321)
I (012), C (0132), N (0133), P (0112)	L (3432)	3x(1133)	J (1232)
I (012), J (0123), N (0133), P (0123)	L (3432)	3x(1133)	C (2112)
I (012), J (0123), N (0133), P (0112)	L (3432)	2x(1133), (1333)	C (2112)

Table 3. Piece categories for possible solutions with L in the centre.

Finally, for completeness we must consider the case where the L is in the centre. It turns out this does not lead to any solutions. There are many more options for the piece categories, shown in Table 3. These can all be checked for solutions by looking at the piece pairings, drawing diagrams analogous to Figure 6, and attempting to finish the tetrahedron. Some rows can be easily eliminated as possible solutions, because it must be possible to group the 1's and 3's in sets of three associated with a vertex or face, respectively. For example, the second row requires that both 1's in the free N (1321) be touching the same vertex, which is impossible.

Other puzzles

This analysis technique is useful for other polysphere puzzles with a symmetrical target shape. The reader is invited to try it on the puzzles shown in Table 4.

No.	J	C	N	P	D	Y	L	Z	T	O	D3	L3	C3	I3	I2	Solutions
0	1	1	1	1			4							1		1
1		4					1					5				2
2							6							3	1	4
3	1	1	1	1			2	1	1					1		17
4	1	1	1	1		1	1	1	1				1			2
5	1	1	1		1	1		1	1	1	1					1
6	1		1	1	1	1		1	1	1	1					1
7	1	1	1		1	1	1		1	1	1					1
8		2					2							1		1

Table 4. Puzzles for the 35-ball tetrahedron or 19-ball octahedron (#8).

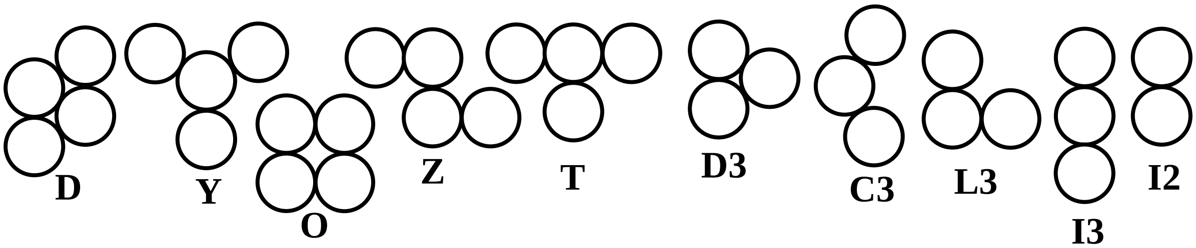

Figure 7. Additional piece shapes used in Table 4.

In Table 4, #0 is the Giant Pyramid puzzle, and #1-2 are from [1]. Puzzles #3 and #4 are modifications I found of the Giant Pyramid puzzle. For these puzzles piece N does not end up in the centre (Table 3 may be useful). Puzzles #5-7 were found using BurrTools [3]. They are the only 35-ball tetrahedron puzzles with the following properties: (1) nine planar pieces, none larger than 4-ball, (2) no linear pieces, (3) all pieces different, and (4) a unique solution. Try this analysis technique when the target shape is an octahedron (puzzle #8) or on a box packing puzzle [1].

References

[1] Leonard Gordon, Some Notes on Ball-Pyramid and Related Puzzles, 1986. For a downloadable pdf, see http://www.gibell.net/puzzles/
[2] Kadon Enterprises, http://www.gamepuzzles.com
[3] BurrTools, http://burrtools.sourceforge.net/