



Figure 1. Stewart Coffin's *Ball Octahedron,* made by Wayne Daniel (photo courtesy Nick Baxter) and a ball pyramid puzzle available on Amazon.

My puzzle collection includes many ball puzzles, one example is Ball Octahedron, design #232 by Stewart Coffin (Figure 1, left). This puzzle is expertly made by Wayne Daniel, and includes non-planar pieces which are difficult to make accurately. The puzzle on the right I do not own, but the (unmodified!) photo is from the Amazon product page. This puzzle pyramid shows certain "geometric anomalies" which suggest the fit is poor. Since this puzzle is not interlocking, some may think a good fit isn't important. But I disagree— a ball pyramid puzzle with poorly made pieces is confusing. The subtle differences in angles are hard enough to understand with a well-made puzzle, and badly made pieces only add to the confusion. Unfortunately, in my experience, poorly made ball pyramid puzzles are common.

#### Manufacturing Techniques for Ball Puzzles

Making ball puzzles isn't rocket science. Nonetheless, inexpensive puzzles often seem to have been made haphazardly. The first problem is obtaining balls which are perfect spheres. For some materials (e.g. acrylic) this is not a problem, but wood balls are often imperfect spheres. Even detecting out-of-round balls is difficult.

Ball pyramid puzzles usually require a base plate or they will fall apart. The simplest base plate is a tray—this only supports the bottom pieces at the edges. An improvement is an indentation or hole at the location of each ball in the bottom layer. This provides support for each bottom piece, but the holes must be located with great accuracy (I have several puzzles where the spacing of the indentations is off). The ideal base plate is laser-cut (Figure 5), because high accuracy can be achieved.

For wood puzzles, the most common way to join balls together is to drill holes in each ball and glue in a dowel. Done properly, this produces a very strong joint which can still flex slightly. A subtle point is that when you push the joint together, the balls tend to become **too close** together. In general, if you can't see the dowel, the balls are probably too close together (Figure 2). One way to ensure that this does not happen is to use the base plate itself. If you rest the part in the base plate while the glue is drying, the balls will be the correct distance apart.



# Figure 2. Two doweled balls pushed too close together, two balls glued together to form a "bi-sphere", (much larger) a design for a connector to be 3D printed.

Other difficulties with doweled joints are drilling holes through the centre of the balls, and getting the angles between three balls correct. For this a drill press and special jigs are recommended. Errors in the angles or ball separation often lead to defects in the final pyramid shape (as in Figure 1, right). If you drill holes the same diameter as your dowels, there may be insufficient space for glue and the joints can eventually come apart. Despite these complexities, properly executed doweled joints are probably the best way to join wooden balls.

In 2002, Jan de Ruiter adopted a novel technique to make his *Hollow Pyramid* [1] exchange puzzle for IPP22 in Antwerp. He had a "double ball chisel" made from the outline of a bi-sphere (middle of Figure 2). A wood turning company used this special chisel to produce thousands of perfectly shaped bi-spheres on an automated lathe. Most pieces were made by drilling and doweling together two bi-spheres, which saved a lot of gluing joints (see Figure 3).

An alternative technique for connecting balls is to glue them together directly (Figure 2). This produces a joint which is weak as well as ugly; such pieces will often break falling on a hard floor. The advantages of this method are that the ball spacing is perfect, and correct angles are obtained by placing the pieces in the base plate. For some materials (glass or metal, for example) drilling holes is difficult and gluing directly is the only real option. The best glued joints I have seen are in Tetra Tops [2], toy tops made from acrylic spheres (Figure 3); these glued joints are practically invisible (the glue used is clear and applied only where the balls touch).



Figure 3. Left to right: Tetra Tops, L. Gordon molded polyspheres (d = 22 mm or 7/8"), *Hollow Pyramid* bi-sphere pieces (d = 20 mm), my wood pieces (d = 5/8") with 3D printed connectors, Kubi-Games molded pieces (d = 18 mm).

One way to increase the strength and attractiveness of glued joints is to glue a thin washer or ring between pairs of balls. I have not found a ready-made part which works well for

this, one option is a 3D printed connector, as shown in Figure 2. This part perfectly matches the sphere curvature, giving good glue contact and producing a very strong, albeit hybrid piece (see the pieces in Figure 3).

Another alternative is 3D printing the entire part (either solid or hollow). This produces very accurate angles between balls, even for non-planar pieces. The disadvantages are the expense, and spheres tend to show stair-stepping when printed in layers. I have made two of my IPP exchange puzzles this way (*Screwy Octahedron* for IPP30, and *Octetra* for IPP33). Both these ball puzzles are interlocking, and the strength of the material was an important consideration.

Injection molding is an inexpensive way to create plastic puzzles, but it is only economical in very large quantities. Molded plastic polyspheres have been made by the Germany company **Kubi-Games** [3] (Figure 3), as well as commercial puzzles by **Lonpos** and **Educational Insights**. Back in the 1980's and 90's, Leonard Gordon used home-made molds and poured polysphere pieces from a clear plastic resin (Figure 3). These parts are accurate as well as durable and attractive [4]. The disadvantage is the non-trivial task of making molds, and dealing with air bubbles and other contaminants.

If you have two pieces with a 90° angle, you can test their accuracy by putting the right angles together to make a 6-ball octahedron. Any inaccuracies in the pieces will show themselves as inaccuracies in the shape of this 6-ball octahedron. Laser-cut base plates (Figure 5) can also be used to gauge the accuracy of planar pieces.

#### Geometric Constraints on Ball Puzzles and their Base Plate

Let **d** be the ball diameter, and **c** the connector diameter. The base plate has thickness **t** and is composed of holes of diameter **h**. If the base plate has indentations instead of holes, **t** and **h** are the depth and diameter of the spherical indentations. Figure 4 shows these parameters on a typical piece (bi-sphere).

For everything to work nicely, the connectors must not touch one another [5], the connectors must not touch the base plate, and the balls must not touch the table [6]. These rules can be stated as three mathematical inequalities (Figure 4). Once the ball diameter **d** is specified, these inequalities constrain the choice of connector diameter **c**, base plate thickness **t** and base plate hole diameter **h**. An example of their use will be given in the next section. Note that if your puzzle does not need a base plate only the first constraint applies.



Geometric constraints:  
1. 
$$c < d/2$$
  
2.  $t < (d-c)/2$   
3.  $h < 2\sqrt{t(d-t)}$ 

Figure 4. Geometric constraints for ball pyramid puzzles and the base plate.

The geometric constraints provide upper bounds for **c**, **t** and **h**, but these three parameters must also not be too small. If the connectors are too thin, the pieces will break easily, and if **t** or **h** are too small the base plate will not provide enough support for the pieces. As a rule of thumb, the three parameters should not be smaller than about  $\frac{1}{2}$  of the upper bounds given in Figure 4. If you have a base plate design, you should use material with thickness  $t > \frac{1}{2}(d - \sqrt{d^2 - h^2})$ .

## Making your own Ball Puzzles

Commercially available ball puzzles generally assemble into only one shape. I wanted to create a library of pieces large enough so that I could try almost any ball pyramid puzzle. I didn't care what the pieces looked like or how durable they were.

I purchased 400 5/8" (15.88 mm) wood balls for \$20, and designed eight different lasercut base plates (laser cut by Ponoko [7], Figure 5). At first, I glued the balls using PVA wood glue, later I used specialized 3D printed connectors (Figure 3). Although these home-made pieces are small, ugly and fragile, their fit is consistently better than most commercially-made ball puzzles I own. The cost of my 100 home-made pieces plus base plates is less than \$40. If the 100 pieces were 3D printed the same size (hollow) by Shapeways, their cost would be over \$200.



Figure 5. Some laser-cut base plates for 5/8" pieces. The rightmost plate is a jig for creating pieces with angles found in hexagonal sphere packing (see [10]).

For my home-made pieces, d = 15.88 mm so the geometric constraints (Figure 4) suggest a value of **c** between 4 mm and 7.94 mm. I use "fat" 3D printed connectors with **c** = 7.88 mm. The suggested base plate thickness is then 2 mm to 4 mm, I use laminated bamboo sheets with **t** = 2.7 mm. The third geometric constraint gives a suggested hole diameter **h** of 6 mm to 11.93 mm, I use **h** = 11 mm.

While you can easily find hundreds of burr designs on the web, there seem to be few polysphere puzzle designs available in print or online. Bernhard Weizorke wrote a nice introduction to polysphere puzzles in CFF25 [8], and his "Compendium of Polysphere Puzzles" contains over 50 designs [9]. The latter document is difficult to find these days. In 1986, Leonard Gordon wrote "Notes on Ball-Pyramid and Related Puzzles", this document is filled with designs and ideas. I have taken the liberty of scanning this document; it can be downloaded from my web site [10].

### References

- [1] Jan de Ruiter, Hollow Pyramid Solved!, CFF 32, August 1993.
- [2] Tetra Tops were made by Duncan, you can find information on them at <u>http://www.tetratops.com</u> A few years ago, I bought four on eBay.
- [3] Kubi-Games, http://kubi-games.com/
- [4] Kadon Enterprises, http://www.gamepuzzles.com/
- [5] George Bell, Classification of Polyspheres, CFF 81, March 2010.
- [6] George Bell, Pyradox: A Pyramid Packing Paradox, CFF 94, July 2014.
- [7] Ponoko, https://www.ponoko.com/
- [8] Bernhard Wiezorke, Puzzling with Polyspheres, CFF 25, 1991.
- [9] Bernhard Wiezorke, Compendium of Polysphere Puzzles, 1996.
- [10] Leonard Gordon, Some Notes on Ball-Pyramid and Related Puzzles, 1986. For a downloadable pdf, see <a href="http://www.gibell.net/puzzles/">http://www.gibell.net/puzzles/</a>