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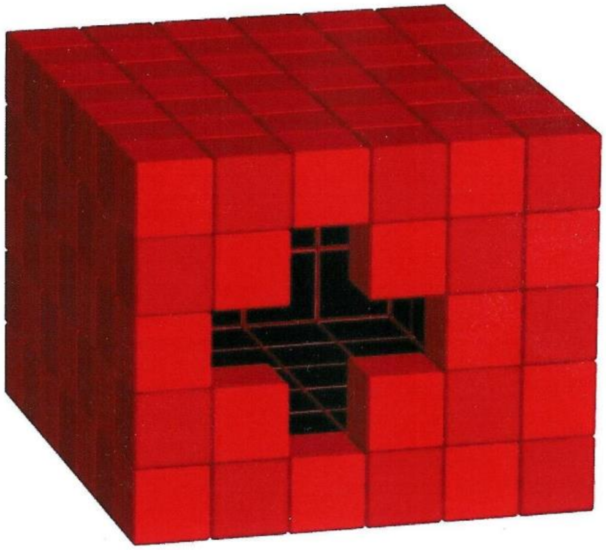

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Pentomino Case Puzzles
by George Bell

Since Sol Golomb described pentominoes in 1953 [1], they have become ubiquitous puzzle pieces. Sixty-two years later, new pentomino puzzles are still being invented. Recently, I found some new pentomino puzzles which are simple in concept, but can be devilishly difficult to solve, even with the aid of a computer.

The reason these puzzles can be difficult is that the solutions involve rotations. We will consider only "fully legal" piece rotations. These are rotations which can be performed using exact size pieces in exact size boxes.

Pentomino Case Puzzles

We use Golomb's notation [1, 2] to refer to the twelve pentominoes: F, I, L, N, P, T, U, V, W, X, Y, Z. A set of pentominoes has a total area of 60 units and can form a tremendous variety of 2D shapes [3]. We'll consider pentominoes made from cubes, so their total volume is 60 voxels—these pieces can also form many 3D shapes. The simplest 3D shapes are the three boxes: 2×3×10 (12 solutions), 2×5×6 (264 solutions) and 3×4×5 (3940 solutions). My idea was to create a box enclosing the pieces with a restricted opening, such that all 12 pieces can be packed inside.

The most interesting box is 3x4x5, because it has the greatest variety of solutions, and because many piece rotations are possible. The best place for the opening is one of the 4x3 faces, for if the opening is in any other face, removal of the I pentomino will require a minimum of a 1x5 opening.

At this point we concentrate on one opening in a 4x3 face. Suppose we consider an opening of area 5, it must have the shape of a pentomino. (By now, we should not be surprised by yet another appearance of the pentomino!) There are 32 possible "pentomino openings", because most pieces have multiple orientations in a 4x3 tray.

BurrTools [4] finds that only a single case is solvable without rotations (Figure 1), the X pentomino. In the assembled puzzle, no piece can lie in the plane of the front face, except for the X pentomino itself. In addition, the L, N, and Y pentominoes can't be found in certain orientations, because they will be unremovable (for L and Y, even allowing rotations). These clues may be useful in finding a solution by hand.

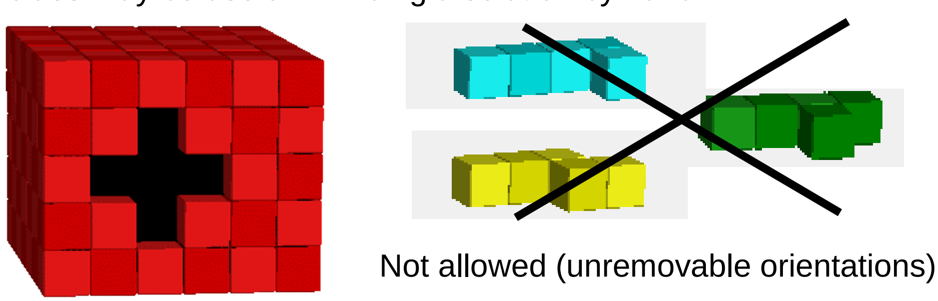

Figure 1. The red *First Aid Pentomino Case*, no rotations required.

I made a prototype of the puzzle in Figure 1 using 15 mm LiveCubes [5], and immediately discovered a problem. The difficulty is that the pieces can be hard to manipulate inside the box. Certain pieces have a frustrating tendency to rotate after insertion. In order to make it less of a dexterity challenge, the puzzle should be as large as possible, at least big enough to get fingers inside the opening. Additional holes in other faces are also useful for piece manipulation. Other improvements are making the box transparent and much thinner than the pieces. A LiveCube box, being opaque and the same thickness as the pieces, is not ideal.

I made a second prototype starting from a commercial puzzle: *Katamino Deluxe* [6]. The pieces are high-quality wood pentominoes based on a 2 cm cube size; many of the pieces appear to be made from a single piece of wood. I designed a clear acrylic box to be laser cut. The box faces are attached using machine screws, nuts, and “T-cutouts” with the 3x4 faces being removable. I found plans for making such boxes here [7]. I added fourteen 27 mm diameter holes in the other faces of the box to assist in manipulating the pieces. These holes seem quite large but do not allow a 2 cm cube to be removed (however, they do allow certain “illegal rotations”, which we won’t use). The laser-cutting plans are shown in Figure 2, contact me if you would like a copy of them. The finished puzzle is shown in Figure 4.

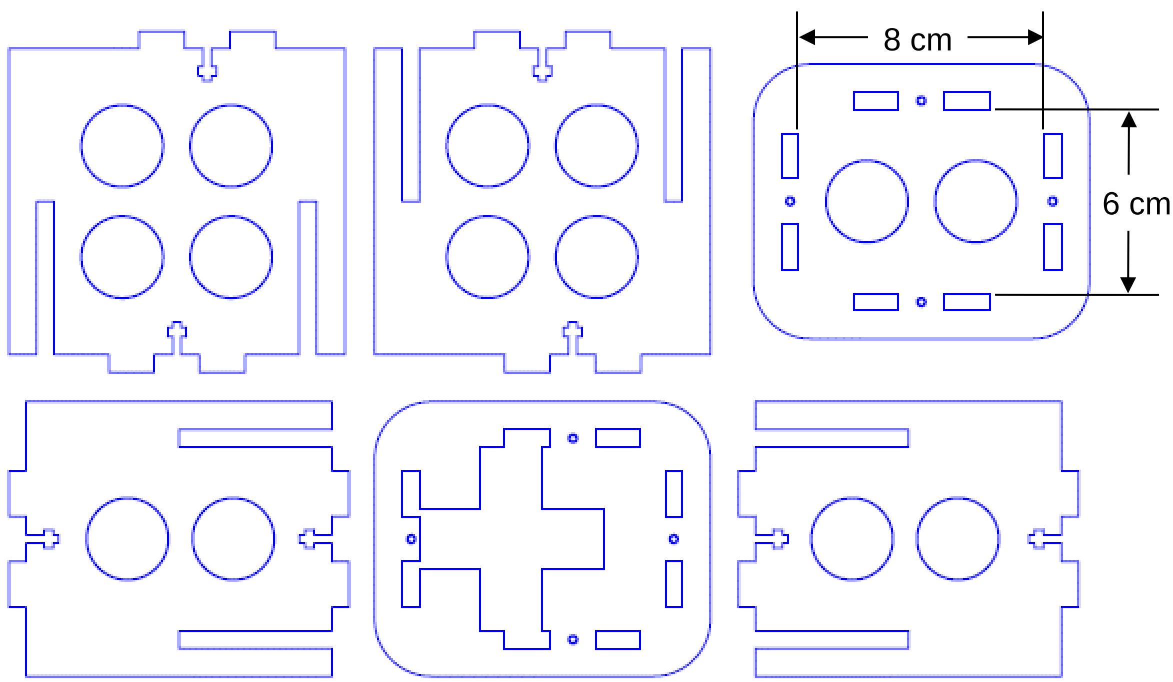

Figure 2. Laser-cutting plans for the *Red Pentomino Case* (not actual scale).

Only the X-pentomino opening is solvable without rotations, but what other “pentomino openings” are solvable with rotations? To my surprise, it turns out that almost all the 32 cases are solvable with rotations. In fact, we can reduce the opening down to a 1x3 rectangle, as shown in Figure 3. All four 1x3 boxes are solvable (with rotations), and the solutions are completely different.

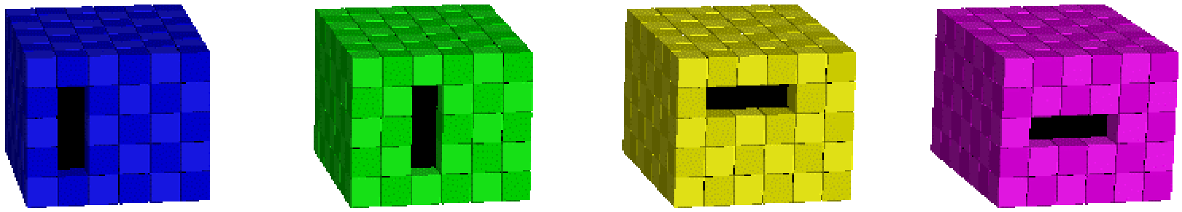

Figure 3. The four 1x3 *Pentomino Case* puzzles: blue, green, yellow, purple.

The puzzles in Figure 3 cannot be solved by BurrTools because they involve rotations. The rotations are not difficult to perform, aside from the fact that they must be done inside a box. Often, the difficulty is not in moving or rotating a particular piece, but keeping other pieces from shifting or rotating out of position. A strategically placed finger is often useful to block a recalcitrant piece.

case	piece removal order	# rotations	known solutions
red	{V, N}, {T, L}, Z, P, U, Y, W, F, X, I	0	3 (w/o rotations)
blue	{L, N, V}, P, T, I	4	8 (all similar)
green	{U, X}, T, I, L, V, Y, Z	4	1
yellow	I, V, N, F, Z	6	2 (both similar)
purple	{U, X}, T, Z, N, V, F, I	4	1

Table 1. Piece removal order (before rotations) for one solution to the cases in Figures 1 & 3. Pieces in brackets can come out together.

Table 1 shows the order in which pieces which come out (before rotations) for one solution to the puzzles in Figures 1 and 3. The blue case has 8 solutions which are minor variations of one another. The green case has a legal solution, and also an "almost solution" where 11 pieces can be removed from the box, but the final T-pentomino cannot be removed (unless an illegal rotation is used).

I was surprised to discover that the start of the solutions in Table 1 can be displayed by BurrTools [4] using the "group pieces" feature (which I had never used). Simply group all pentominoes **not** shown in the second column of Table 1 with the empty box. After solving for the disassembly, the start of the solution will be shown, up until the rotations. Note that this method may also find false solutions where the remaining grouped pieces cannot be removed. This is not a useful technique to search for solutions, because you don't know beforehand which pieces to group.

Although Table 1 describes the start of each solution, the rotational moves are left for the reader to puzzle out. This is difficult—feel free to contact me for a copy of my notes detailing how to execute each solution (both assembly and disassembly).

Summary and other ideas

We have considered a *Pentomino Case* with a single opening in one 3x4 face. With a 3x1 opening, all four cases are solvable. All these puzzles probably have multiple solutions, but it is not easy to find all solutions since several piece rotations are involved.

Figure 4. A prototype *Pentomino Case* made from 5.6 mm clear acrylic and machine screws. Wood pieces are from a *Katamino* set [6].

The prototype puzzle (Figure 4) has five different lids which can be swapped out. Removing the pieces is much easier than finding a way to get them all back inside. The case makes a nice storage unit for a set of pentominoes. If you can't get all 12 pieces back you can always use a screwdriver to remove and replace the lid.

It may be interesting to consider disconnected openings, and/or openings on other faces. The round holes in the Figure 4 case enable certain "illegal rotations", and it may be interesting to take advantage of these. One could also use openings which are not aligned with the grid, or are not composed of squares.

References

[1] Sol Golomb, Polyominoes: Puzzles, Patterns, Problems, and Packings, 2nd edition, Princeton University Press, 1996
[2] https://en.wikipedia.org/wiki/Pentomino
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[4] BurrTools, http://burrtools.sourceforge.net/
[5] LiveCube, http://www.livecubeshop.com/
[6] http://en.gigamic.com/game/katamino
[7] How to make anything (using acrylic and machine screws), http://tinyurl.com/3me672n