Designing the Trapdoor Octahedron puzzle

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My G4G15 exchange is the **Trapdoor Octahedron** puzzle (Figure 1), a 3-piece **coordinate motion** puzzle. Coordinate motion refers to the fact that to come apart, all three pieces must move at the same time.

Figure 1. Trapdoor Octahedron puzzle, 3D printed.

The puzzle is not difficult to take apart, but most will struggle to reassemble it. In cryptography, a trapdoor function is one that is easy to compute in one direction yet difficult in the opposite direction (finding its inverse). Thus, the **Trapdoor Octahedron** puzzle is a mechanical analog of a trapdoor function.

Coordinate motion puzzles are difficult to design, as the pieces must work together to constrain their movement. Any change will generally ruin the puzzle. This document outlines the design process for this puzzle—the last page gives the solution.

Step 1: The basic piece

Coordinate motion puzzles often have identical pieces. This is not a requirement—in fact the **Trapdoor Octahedron** uses three different pieces! However, it is a good starting point in the design of a coordinate motion puzzle.

I wrote a program to search for dissections of the rhombic dodecahedron into three identical pieces. The program also requires that the dissection be symmetric about the 3-fold axis of symmetry. See [1] for details. The piece shown in Figure 3 is one out of 231 found by my program.

An easy way to build the basic piece uses a special building block, the green pyramid in Figure 2. The green pyramid can be made from a rhombic dodecahedron by cutting along every triangle connecting an edge to the center of the polyhedron. This dissects the rhombic dodecahedron into 12 identical pyramids, one for each face. In BurrTools [2], this green pyramid is four voxels in the “Rhombic Tetrahedra” geometry; Stewart Coffin calls it a “Rhombic Pyramid Block” [3]. We will also use this pyramid cut in half lengthwise (orange in Figure 2).

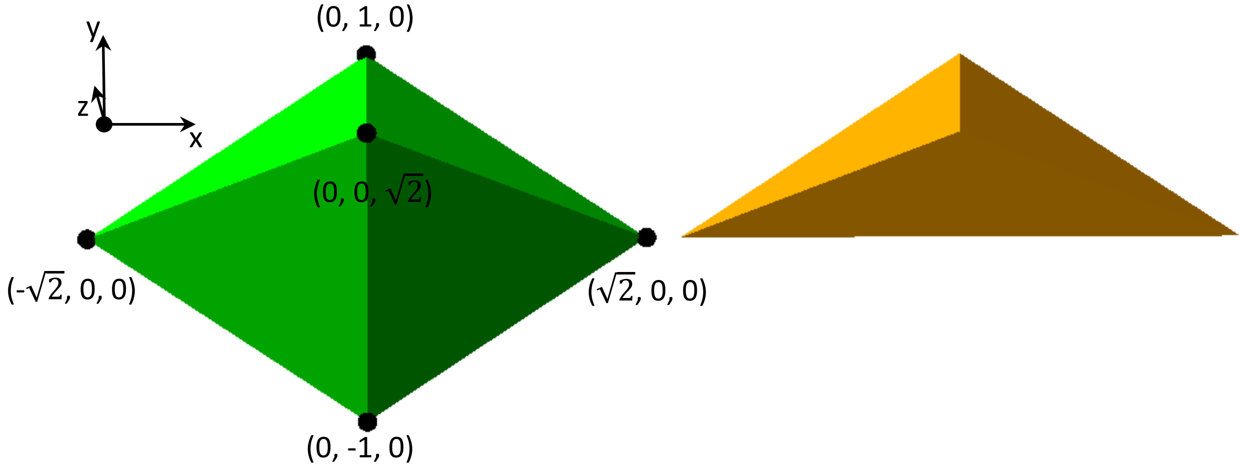


Figure 2. Left: the building block (green); right: cut in half lengthwise (orange).

We now combine three green pyramids and two orange halves to make the basic puzzle piece (Figure 3). Each basic puzzle piece has the volume of four pyramids, and three copies form a rhombic dodecahedron. This basic piece has 180° rotational symmetry about the z-axis (coming out of the page).

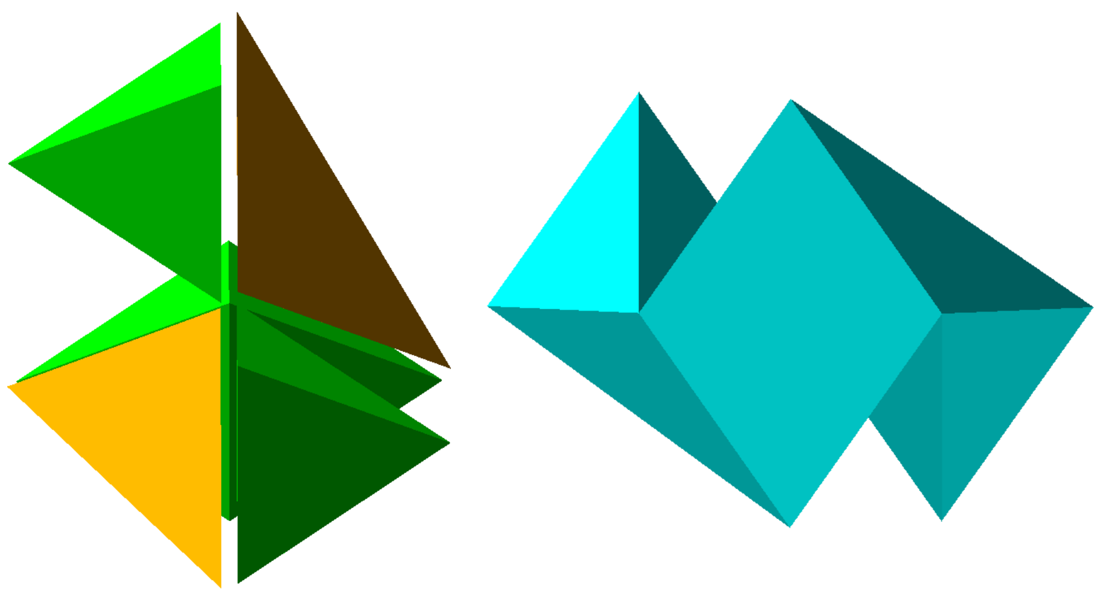


Figure 3. Three pyramids (green) and two half-pyramids (orange) form the basic piece. Left: pyramids slightly separated; right: completed piece (bottom view).

Step 2: Partial stellation into an octahedron

Three copies of the basic piece assemble via coordinate motion to form a rhombic dodecahedron. Coordinate motion means that all three pieces must move at the same time. This process is shown in Figure 4 (left, looking down the 3-fold symmetry axis). The assembly feels loose, because there is more than one way for the pieces to go together (the puzzle is “CM+” as defined in [4]).

One way to fix this looseness is to stellate the rhombic dodecahedron, which adds twelve green pyramids to the outer faces. To remain a three-piece puzzle, we need to connect the added pyramids to the existing pieces, and there are many ways to do this. I have chosen only a partial stellation, adding four green pyramids. This results in a non-regular octahedron.

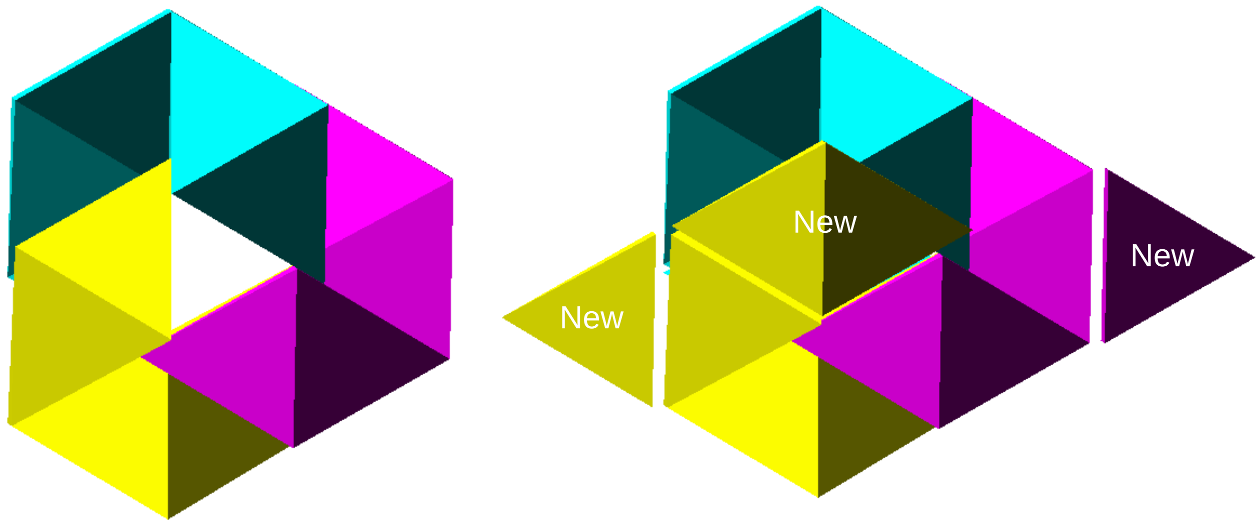


Figure 4. Left: coordinate motion assembly; right: same view showing the added pyramids (marked “New”). The 4th (aqua) pyramid is hidden behind.

This partial stellation is the most important step in the design because the added pyramids restrict the piece motion so that the puzzle comes apart in only one way. In addition, after partial stellation the three pieces are all different. The assembled shape is a non-regular octahedron (Figure 5, right). Each face is not an equilateral triangle but an isosceles triangle with odd angle from the rhombic dodecahedron face, 70.53°.

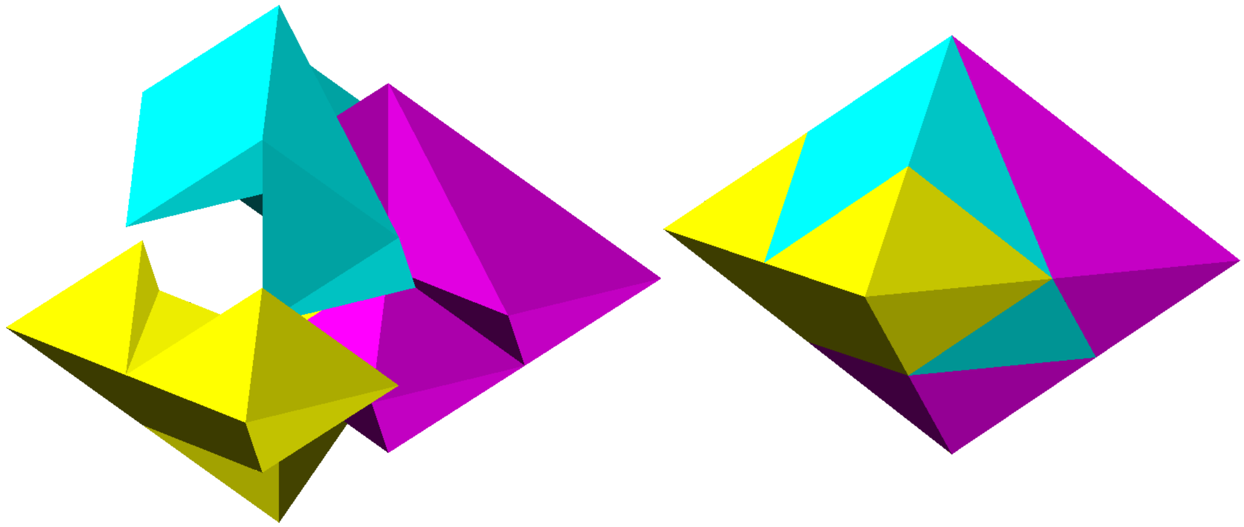


Figure 5. Top view of the puzzle being assembled, final assembly into a (non-regular) octahedron.

Step 3: Vertical stretch into a regular octahedron

If we stretch the octahedron along its short axis by a factor of $\sqrt{2}$ it becomes a regular octahedron. Remarkably, the coordinate motion is unchanged by this transformation. The reason is that the pieces have planar faces and move linearly without rotation (see [1]).

Step 4: Truncation

The final step is to truncate the octahedron. The truncation makes it easier to 3D print, plus I find it aesthetically pleasing. Figure 6 shows the three pieces 3D printed.

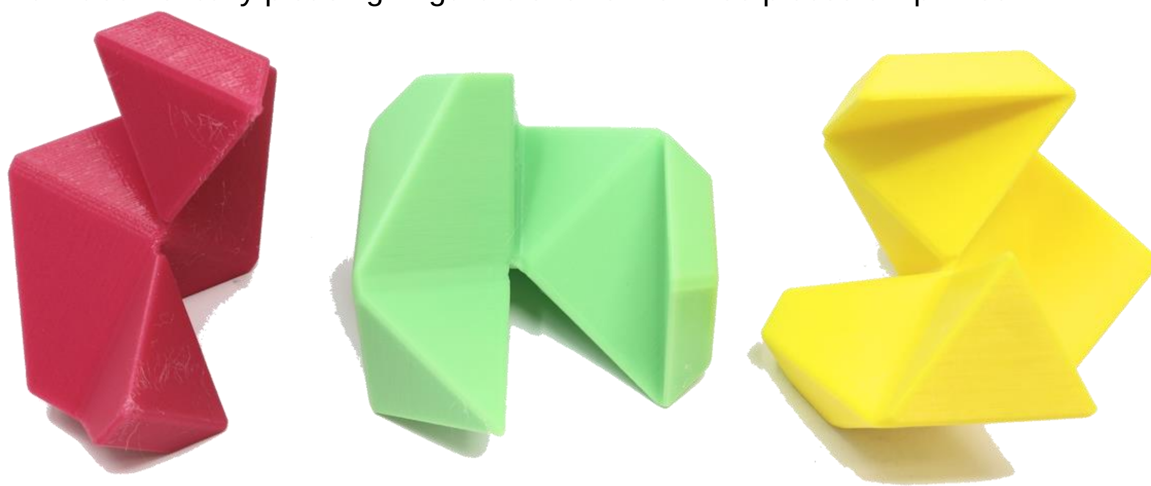


Figure 6. The final **Trapdoor Octahedron** pieces 3D printed (normal version). All pieces are now different, and all have different volumes.

The dimensionless units in Figure 2 are multiplied by a scale factor to get the final puzzle size. The scale factors used by the two versions are given in Table 1. The exchange version is the small size. The puzzle was designed using BurrTools [2] and OpenSCAD [5]. You can 3D print your own copy at Printables [6].

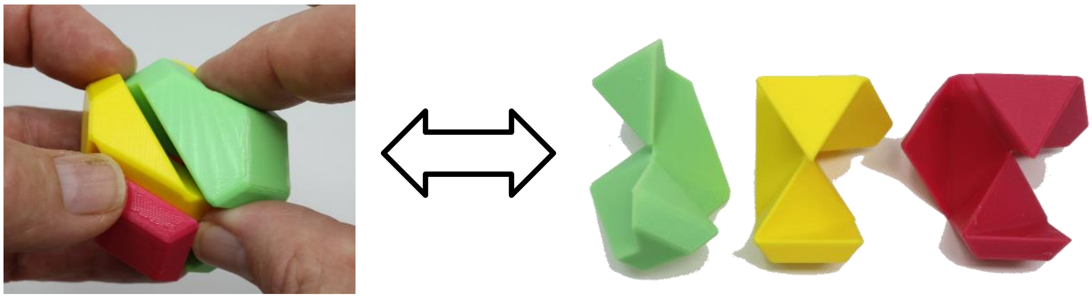
version	scale factor	BurrTools [2] Unit Size	Bevel	Offset	edge length of rhombic dodecahedron	regular octahedron	truncated octahedron
normal	15.59	13.5	0.500	0.05	27	54	18
small	11.67	10.1	0.375	0.05	20.2	40.4	13.47

Table 1. Sizing numbers for the final puzzle (all dimensions in mm).

References

[1] George Bell, Coordinate Motion Puzzles, Part 3, CFF 120, March 2023, p. 10–15.
[2] BurrTools, http://burrtools.sourceforge.net/
[3] Stewart Coffin, Geometric Puzzle Design, A.K. Peters, 2007, p. 103.
[4] George Bell, Coordinate Motion Puzzles, Part 1, CFF 118, July 2022, p. 16–21.
[5] OpenSCAD, https://openscad.org/
[6] https://www.printables.com/model/693304-trapdoor-octahedron-puzzle

Trapdoor Octahedron Solution

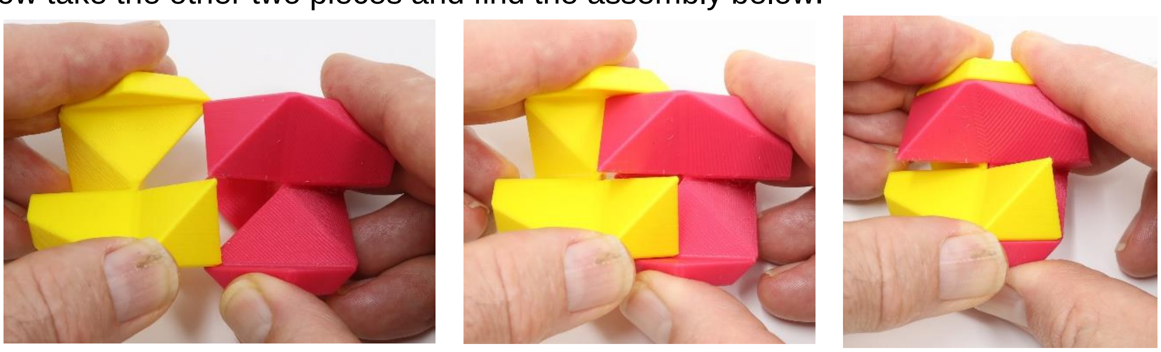


Designer: George Bell
Goal: Take it apart and put it back together
Solution video: https://www.youtube.com/watch?v=6pLlXKmy0CU

Disassembly:
Note that two of the hexagonal faces contain a triangular face of a different color. To make the puzzle expand, press in these two faces with your thumb and forefinger (as shown above). Use your other hand to stop the motion. If you expand it far enough, it will fall apart.

Assembly: This is much harder. First, identify the smallest piece. This piece is also the only one with 180° rotational symmetry. In the above photos, it is the green piece—find this smallest piece in your puzzle (it likely has a different color).

Now take the other two pieces and find the assembly below.



The yellow piece slides in from the left, the red piece from the right. In the right photo, these two pieces are in their assembled configuration. We now need to somehow get the green piece between them.

To get the green piece in there, back the red and yellow pieces up until you are in the position of the left photo. Note that the two central points, as well as those above and below, should be just touching. With the red and yellow pieces in this position, add the green piece from above. There are four points which must engage into grooves. If you can engage all 4 simultaneously, the three pieces will slide together.